

Should we discount by project or by company?

A story told by Santa Claus.

In corporate finance a major problem is project risk and how it influences the discount rate. Should we discount each project with a different discount rate or should we use one discount rate for the whole company? This question becomes particularly interesting in the case of license contracts where usually a company with a high cost of capital negotiates with a company with a low cost of capital. Is the project simply worth more because it is in a big company's hands? Or is this all wrong?

Discount by project!

Brealey, Myers, and Allen¹ claim that the project values must add up and that the value of a company equals the sum of the values of its projects. They claim that each project can be valued as if it were a mini-company. If the company holds two projects then – according to them – the value of the company can be described as follows:

$$V(A_1 + A_2) = V(A_1) + V(A_2) \quad (1)$$

They even say “The company cost of capital is defined as the expected return on a portfolio of all the company's securities. It is (...) the appropriate discount rate for the firm's average-risk projects.” If a project is more or less risky than the company's existing business, then it has to be discounted at a different rate. By the way, the company's cost of capital is assessed using the CAPM². They also give a reasonable justification: It would be

unfair to discount a risky project at the same rate like a relatively safe project.

No, discount by company!

In the theory outlined above only the company cost of capital is observable to some degree, because only securities on the company are tradable. You typically cannot invest in projects, and money is raised by the company as an entity. Obviously there is a certain purpose linked to the raised money, but the money stays within the company, not in the project. Furthermore, a company that has more stable cash flows has a better risk profile than a company that has cash flows that may vary a lot. The more diversified a company, the more stable its cash flows; if one project underperforms, there might be another project that compensates. This idea works both in the cash flow perspective and the value perspective. So we would expect a lower cost of capital from a diversified company. We therefore naturally expect:

$$V(A_1 + A_2) \geq V(A_1) + V(A_2) \quad (2)$$

This means that the value of a portfolio of assets is superior to the value of its parts. This effect is called superadditivity and stems from the diversification effect. We must add here a few restrictions:

1. Of course, the superadditivity only holds as long as the single assets cannot be traded separately; the value of a portfolio of shares equals the sum of the value of the separate stock holdings. No discussion about that!
2. Holding several assets might require some additional costs that wouldn't be necessary otherwise. These administration and coordination

¹ Richard A. Brealey, Stewart C. Myers, and Franklin Allen, „Principles of Corporate Finance“, 9th edition, 2008, McGraw-Hill.

² $r_A = r_f + \beta_A \cdot (r_M - r_f)$

costs might reduce or even offset the superadditivity.

3. If the assets are perfectly correlated, then there is no superadditivity.

Brealey, Myers, and Allen seem to plainly disregard the diversification effect. "More projects" generally means "more diversification" (there might be some exceptions to that rule). The expression "average-risk project" implies that as long as the risk profile is the same, the overall risk does not change; just like in an average calculation. Well, risk is different. It matters, how the projects behave with respect to each other; the so-called correlation. As an illustration we assume companies that have a varying number of projects of the same type. But once the projects are independent of each other, once correlated with 50%, and once completely correlated ($\rho=0$, $\rho=0.5$, $\rho=1$). The projects have a volatility of 50% and a correlation with the market portfolio $\rho_{AM}=50\%$. The market portfolio has a volatility $\rho_M=20\%$.

Table 1: Effect of diversification on volatility and beta.

| | $\sigma_{\rho=0}$ | $\beta_{\rho=0}$ | $\sigma_{\rho=0.5}$ | $\beta_{\rho=0.5}$ | $\sigma_{\rho=1}$ | $\beta_{\rho=1}$ |
|--------------|-------------------|------------------|---------------------|--------------------|-------------------|------------------|
| 1 project | - | - | - | - | 50% | 125% |
| 2 projects | 35% | 88% | 43% | 108% | 50% | 125% |
| 3 projects | 29% | 72% | 41% | 102% | 50% | 125% |
| 4 projects | 25% | 63% | 40% | 99% | 50% | 125% |
| 5 projects | 22% | 56% | 39% | 97% | 50% | 125% |
| 100 projects | 5% | 13% | 36% | 89% | 50% | 125% |
| >>100 | 0% | 0% | 35% | 88% | 50% | 125% |

Even though all projects have exactly the same risk profile it becomes obvious that

one additional project changes the risk profile of the company. Risk hence does not work the same way like averages. The idea of "average-risk projects" is non-sense (except in the case of perfect correlation).

We clearly recognise the effect of diversification, which naturally is stronger the weaker the correlation. And very importantly, it is this volatility that you can observe on the stock market³. You don't observe a project volatility because you can only observe volatilities of traded securities. Consequently, the diversification is also included in the observation. But what does that mean for a project valuation? How much is a project worth?

Project Value vs. Company Value

A project on its own does not offer diversification. But depending on the context, in which the project is developed, i.e. depending on the company who finances that project, the risk profile might be improved by other projects. So, the risk profile an investor faces is not the same if the project is developed in a one-project company or in a company with a whole portfolio of projects. The actual value of the project A in a company C can be expressed as follows:

$$V(A) = V(C \text{ with } A) - V(C \text{ without } A) \quad (3)$$

The value of the project A corresponds to the value added by the project to the company.

Let us see what this means for some easy cases like the companies we have just analysed.

³ with all caveats like liquidity, special events, etc.

Table 2: Cash flows of project A.

| | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
|-----------|-----|-----|----|----|----|----|
| Project A | - 1 | - 1 | 2 | 2 | 2 | 2 |

We take a look at companies that have projects of the type as displayed in table 2. Their volatility is assumed to be 50%. With the same assumptions we receive the following interesting valuation results.

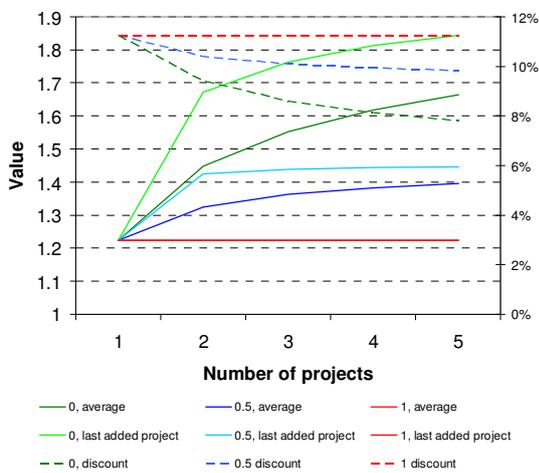


Figure 1: Project value depending on company.

The figure displays for each of the three correlation scenarios two project values. One is the average value, which is the company value divided by the number of projects. The second is the value of the last added project, i.e. the value another project adds to the company when going from n-1 projects to n projects. Naturally, we expect that the value of the company increases over-proportionally because the discount rate decreases with increasing diversification (except if there is none with $\rho=1$). Consequently, the average value of a project increases. The value of an additional project also keeps increasing because each time we also attribute the value increase of the other projects due

to the better diversification to the new project⁴. This definition overstates in a way the project value. Take the case of a 2-project company with identical projects ($\rho=50\%$). Obviously each project is worth the same, a third of the company value of 2.65, i.e. 1.32. Nevertheless, the value increase from a company with two projects to a company with three projects is 1.42. But $2*1.42=2.84>2.65$. This is a little disturbing. Wouldn't that mean that the sum of all project values is greater than the company value, contradicting our claim (2)? No, because claim (2) states that the company value is greater than the sum of each project alone, i.e. not in the company. But when you consider investing in a new project, it makes sense to account for the effect of the new project on the overall risk profile of the company. And here we also get back to Brealey, Myers, and Allen's justification that it would be unfair to discount a risky project at the same rate like a safe project. Imagine the following situation: Project A_1 has a volatility of 50% and project A_2 has a volatility of 80%, but the cash flows are exactly as described in table 2. The valuation results are as follows:

Table 3: Valuation of A_1 and A_2 .

| | A_1 | A_2 |
|---------------|--------|-------|
| β | 125% | 200% |
| Discount rate | 11.25% | 15% |
| Value alone | 1.22 | 0.83 |

⁴ Note that equation (2) and (3) do not contradict each other, because in equation (2) $V(A)$ means the value of project A as a one-project company.

Table 4: Valuation of A₁ and A₂ in a company.

| | $\rho=1$ | $\rho=0.5$ | $\rho=-1$ |
|---------------|----------|------------|-----------|
| β | 110% | 200% | 200% |
| Discount rate | 10.49% | 11.72% | 12.76% |
| V(A1+A2) | 2.63 | 2.34 | 2.11 |
| V(A1) | 1.80 | 1.51 | 1.28 |
| V(A2) | 1.40 | 1.12 | 0.89 |

We clearly see that the riskier project is worth less. This is an interesting observation, as for the company valuation the cash flows, which are exactly the same like for the safer project, get discounted at the same rate. But the value contribution of the riskier project is less as we can see in table 4. This confirms our model.

Another flaw of the Brealey, Myers, and Allen approach is the assumption that apparently all cash flows can be allocated to projects. This is wrong. Some G&A expenses are just there no matter what the portfolio looks like, and taxes are levied on company basis, not on project basis.

Conclusion

When valuing a company from the outside we can apparently rely on the β and cost of capital we observe and use it for all projects. But if we want to compare projects within the company or even consider investments or divestments, then we need volatilities and correlations on project level. Even the Brealey, Myers, and Allen model requires project volatilities. But since they disregard the diversification effect they do not require correlations between projects. The correlation is unfortunately

not quite easy to observe. But as seen, if you want to capture the risk profile of your company correctly, then you must include the correlations.

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