Should we discount by project or by company? (continued)

Last month Santa Claus argued that we should discount by company and never by project. This means that we need to establish a discount rate for the company and then discount each project with the company discount rate. Santa's line of thought was basically the following:

- β_A of asset A is defined as

  $\beta_A = \sigma_A / \sigma_M \rho_{AM}$.
- Diversification reduces volatility σ_A.
- Consequently a diversified company should have a lower beta, which leads to a lower cost of capital.

Diversification increases correlation

But Santa overlooked one very important aspect. While diversification reduces the volatility of a portfolio of assets, it increases the correlation with the market. To illustrate this, think of the market portfolio as the climax of diversification. And obviously, the market portfolio has a correlation of 1 with itself. This increase in correlation exactly offsets the decrease in volatility. Put in other words: reduction of specific risk increases systematic risk, but the risk stays the same.

How β is really defined

Remember that β originally is defined as $\beta_A = \mathrm{Cov}(A,M)/\mathrm{Var}(M)$. Assuming that company A is composed of projects A_1, …, A_n we can develop b and find the following:

$$\beta_A = \Sigma \omega_i \mathrm{Cov}(A_i, M)/\mathrm{Var}(M) = \Sigma \omega_i \beta_i$$

Where ω_i denotes the contribution of each project to the full value of A. So, the beta of the company is a weighted average of the betas of its projects. This leaves us with important questions:

1. Can we therefore value a company project by project and then simply add up the values?
2. What effect has diversification, if any?

Still discount by company!

We still have to discount by company, i.e. with a company discount rate. The reason for this is mathematical. Even though the betas of the projects might add up to the company beta, the discount factors do not (because the cash flows that contribute to the project values are differently timed for each project). As an example imagine two projects; A has a cash flow of 10 in two years with a volatility of 80% and B has a cash flow of 20 in one year with a volatility of 40%. Table 1 displays how the values of the projects and the portfolio (A+B) behave. We have assume $\sigma_M = 20\%$, $\rho = 50\%$, $r_f = 5\%$, and $r_M = 10\%$.

Table 1: Difference between project and company valuation.

	β	Discount rate	Value
A	200%	15%	7.6
B	100%	10%	18.2
A+B	129%	11.5%	26.0

This means that in general the following relation still holds:

$$V(A1) + V(A2) \leq V(A1 + A2)$$

However, it doesn't hold always. In the case of a two-project-company, if the cash flows of the riskier project happen earlier, then the company value can be less than the sum of the project values. In general, later-stage projects are less risky than earlier-stage projects, so the rule usually holds.

Diversification

A very interesting conclusion from the weighted average beta concerns the effect of diversification. Apparently, the dependence between the projects, i.e. how they are correlated between each other, does not have an impact at all on the discount rate. No portfolio manager has to guess how correlated the different projects are. This is quite an interesting observation, since in asset management diversification is seen to be a very useful feature of a portfolio. In the corporate environment this apparently doesn't count.

It is actually not completely true. The tax value of a company, as long as it is not yet profitable, depends on the correlation between the projects[1]. Interestingly one would rather expect the diversification to have an impact on the cost of capital than on the tax value. But math teaches us differently.

[1] Villiger, Ralph (2010) "The Effect of Taxes on the Value of Start-Ups," Journal of Business Valuation and Economic Loss Analysis: Vol. 5 : Iss. 1, Article 6.